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A GLOBAL 1 DEG X 1 DEG ANOMALY FIELD COMBINING SATELLITE, GEOS---ETC(U)

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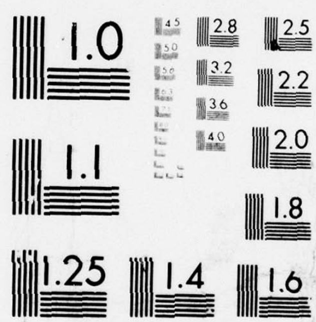

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A GLOBAL $1^\circ \times 1^\circ$ ANOMALY FIELD COMBINING SATELLITE, GEOS-3 ALTIMETER AND TERRESTRIAL ANOMALY DATA

Richard H. Rapp

The Ohio State University
Research Foundation
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This report describes a method for combining satellite derived potential coefficients, mean 1° x 1° anomalies derived from the Geos-3 altimeter data, and mean 1° x 1° anomalies estimated from terrestrial gravity data. The combination procedure is designed to obtain an adjusted set of potential coefficients and an adjusted set of 64800 1° x 1° mean anomalies. The adjusted anomalies can be developed into potential coefficients to as large a degree as is reasonable considering the 1° x 1° data. Such anomalies will be exactly consistent with the

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adjusted potential coefficients.

The method was applied using the GEM 9 potential coefficients, 28176 $1^\circ \times 1^\circ$ anomalies derived from the Geos-3 altimeter data, and 22474 terrestrial anomalies. The potential coefficients were adjusted to degree 12 with the adjusted anomalies being developed into potential coefficients to degree 60. Higher degree solutions in the adjustment process are possible but the computer time needed is quite large.

Foreword

This report was prepared by Richard H. Rapp, Professor, Department of Geodetic Science, The Ohio State University, under Air Force Contract No. F19628-76-C-0010, The Ohio State University Research Foundation Project No. 710335. The contract covering this research is administered by the Air Force Geophysics Laboratory, L. G. Hanscom Air Force Base, Massachusetts, with Mr. Bela Szabo, Contract Monitor.

The work of Kostas Katsambalos, Graduate Research Associate, on techniques for developing $1^\circ \times 1^\circ$ anomalies into potential coefficients, was very helpful for aspects of the research described in this report.

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Introduction

The purpose of this report is to describe a technique for combining information from several different data sources to obtain an improved description of the earth's gravity field.

The specific data which we wish to use in this combination solution are the following:

1. A set of potential coefficients (such as GEM 9 (Lerch et al., 1977)) derived solely on the basis of satellite observations. We can assume that we have a variance-covariance matrix for this data but in practice we will be using a diagonal only form of this matrix.

2. A set of $1^\circ \times 1^\circ$ mean free-air anomalies and their standard deviations based solely on terrestrial measurements. Such a set will not be global in nature.

3. Information derived from Geos-3 (or any other) satellite altimeter. The specific question is exactly what form this information will take. The altimeter data can be processed to yield geoid undulations (neglecting sea surface topography) along the altimeter track (Rapp, 1977a, 1979). These geoid undulations can be used to determine mean undulations and mean gravity anomalies in various size blocks. One could work with either set of values. For our purposes we wish to work with a set of independent values that exist for land areas. Such values are the terrestrial anomalies mentioned in item 2. A possibility would be to use existing data to compute geoid undulations on land, to combine with the altimeter derived undulations in the ocean areas. However, in this case the land undulations would all be statistically correlated and any reasonably rigorous treatment of the data would be practically impossible. We thus decided to use the altimeter implied mean gravity anomalies for one of the basic data sources in the general combination solution.

We now need to consider the goals of the combination method. The method that we choose must yield results that give a consistent representation of our data without sacrificing or losing information that exists within the data. For example, one might visualize a solution for a least squares estimation of a set of potential coefficients using the data items previously mentioned. If the only solution parameters are the potential coefficients to a low degree, (say 15, 30, 40, etc.) we will have not represented all the information in the data set. Yet to carry out a very high degree (180 for example) solution using a rigorous least squares procedure is practically impossible because of the large number of unknowns that are involved.

The method that we will describe in the next section will be designed to meet the objectives stated in the above paragraph.

After developing the theory for this combination solution we will describe various test computations with real data. Our final results will be a set of potential coefficients complete to degree 60 and a set of 64800 $1^\circ \times 1^\circ$ mean anomalies.

The Method

The theory to be used here was originally suggested by Kaula (1966) for the combination of satellite derived potential coefficients and terrestrial gravity data. Deatils of this method can be found in Rapp (1968) with a Fortran program described in Snowden and Rapp (1968). We outline the theory below.

Let $\bar{C}_{\ell m}$, $\bar{S}_{\ell m}$ be a set of fully normalized potential coefficients which occur in the following description of the earth's gravitational potential V:

$$V = \frac{kM}{r}\left[1 + \sum_{\ell=2}^{\infty}\left(\frac{R_B}{r}\right)^{\ell}\sum_{m=0}^{\ell}(\bar{C}_{\ell m}\cos m\lambda + \bar{S}_{\ell m}\sin m\lambda)\bar{P}_{\ell m}(\sin\bar{\varphi})\right] \quad (1)$$

The notation is standard (Rapp, 1977b). (Actually R_B, the radius of the Bjerhammar sphere interval to the earth is usually replaced by an equatorial radius a).

If we are given a set of global mean anomalies, Δg, the potential coefficients (with respect to an ellipsoid of specified flattening) can be computed from (ibid., equation (5 or 6))

$$\left\{\begin{matrix}\bar{C}_{\ell m}\\ \bar{S}_{\ell m}\end{matrix}\right\}_{\overline{\Delta g}} = \frac{1}{4\pi\gamma(\ell-1)\beta_{\ell}s^{(\ell+2)/2}}\cdot\sum_{\ell=1}^{T}\overline{\Delta g}_1\cdot\left\{\begin{matrix}\bar{A}_{\ell m}\\ \bar{B}_{\ell m}\end{matrix}\right\} \quad (2)$$

where:

$$s = \left(\frac{R_B}{R}\right)^2 \quad \text{with} \quad R = \text{an average earth radius}$$

$$\left\{\begin{matrix}\bar{A}_{\ell m}\\ \bar{B}_{\ell m}\end{matrix}\right\} = \iint_{A_1}\bar{P}_{\ell m}\left\{\begin{matrix}\cos m\lambda\\ \sin m\lambda\end{matrix}\right\}d\sigma \quad (3)$$

where A_1 is the block in which the mean anomaly is given and β_ℓ is the averaging operator defined by:

$$\beta_{\ell} = \cos\frac{\psi_0}{2}\,\frac{P_{\ell 1}(\cos\psi_0)}{\ell(\ell+1)} \quad (4)$$

where ψ_0 is the radius of a spherical cap having the same area as the block A_1 (Rapp, 1977c). Note that (2) is a spherical approximation and neglects terrain and other effects as described in Rapp (1977b). Values of the integrals in (3) have been computed by Katsambalos (1978, private communication) using information supplied by Paul (1978, private communication). The principle of the combination solution is simply the comparison of the potential coefficients computed from (2) with those derived from satellite data with an adjustment being performed, recognizing all the data is to be weighted, to obtain a consistent set of potential coefficients and anomalies.

We briefly describe this adjustment process as follows: A general function F is defined:

$$F = F(L_{\ell^a}, L_{x^a}) = 0 \tag{5}$$

where L_{ℓ^a} are the adjusted observations and L_{x^a} are the adjusted parameters. A linearized observation equation is then formed:

$$B_\ell V_\ell + B_x V_x + W = 0 \tag{6}$$

where

$$B_\ell = \frac{\partial F}{\partial L_\ell}, \quad B_x = \frac{\partial F}{\partial L_x}, \quad W = F(L_\ell, L_x^o) \tag{7}$$

where L_ℓ are the actual observations and L_x^o are the observed values of the quantities to be regarded as parameters (e.g. the potential coefficients) of the adjustment. If P_ℓ and P_x are the weight matrices for the observations and parameters, respectively, we have for the correction to the observed parameters, V_x:

$$V_x = -(B_x' M^{-1} B_x + P_x)^{-1} B_x' M^{-1} W \tag{8}$$

with the corrections to the observed quantities (e.g. the gravity anomalies), V_ℓ:

$$V_\ell = -P_\ell^{-1} B_\ell' M^{-1} (B_x V_x + W) \tag{9}$$

where

$$M = B_\ell P_\ell^{-1} B_\ell' \tag{10}$$

In our case we have:

$$F = L_x^o - L_x^c \tag{11}$$

where L_x^o are the given estimates of the potential coefficients (e.g. the GEM 9 coefficients) and L_x^c are the coefficients computed from (2) with the observed set of gravity anomalies. In this case:

$$B_x = I \tag{12}$$

$$[B_\ell]_{PC} = \frac{-1}{4\pi\gamma(\ell-1)\beta_\ell s^{(\ell+2)/2}} \begin{Bmatrix} \bar{A}_{\ell m} \\ \bar{B}_{\ell m} \end{Bmatrix} \tag{13}$$

where the bracket around B_ℓ indicates that the expression on the right side is simply one element in the B_ℓ matrix.

We should note here that (13) applies only for the partial derivatives with respect to potential coefficients. We may also desire to impose information on the spherical harmonic expansion (coefficients $\bar{a}_{\ell m}, \bar{b}_{\ell m}$) of the anomalies such that (for example) the mean anomaly of the adjusted set is zero ($a_{0,0} = 0$); and the first degree terms ($\bar{a}_{1,0}$, $\bar{b}_{1,0}$) are also zero. Equation (13) is then written:

$$[B_\ell]_{\Delta g} = \frac{-1}{4\pi \beta_\ell s^{(\ell+2)/2}} \begin{Bmatrix} \bar{A}_{\ell m} \\ \bar{B}_{\ell m} \end{Bmatrix} \tag{14}$$

which is usually only evaluated for (0,0), and (1,0). Using (12) in (8) we have:

$$V_x = -((B_\ell P_\ell^{-1} B_\ell')^{-1} + P_x)^{-1} (B_\ell P_\ell^{-1} B_\ell')^{-1} W \tag{15}$$

and equation (9) reduces to:

$$V_\ell = P_\ell^{-1} B_\ell' P_x V_x \tag{16}$$

Equation (15) and (16) form the core of the adjustment process. Equation (15) yields corrections to the original potential coefficient estimates while (16) gives the ocrrections to the original anomaly estimates. We point out here that the adjusted anomalies can be developed into potential coefficients using equation (2) to as high a degree as is reasonable. The resultant coefficients will agree exactly with the adjusted coefficients with the higher degree terms (i.e. those above the degree solved for in the adjustment) needed to describe the higher frequency information in the data.

Previous applications (Kaula, 1966, Rapp, 1968) have been restricted to the use of 1654 5° anomalies and potential coefficients up to degree 14. In our applications we intend to use 64800 $1^\circ \times 1^\circ$ anomalies and to adjust as many potential coefficients as possible consistent, however, with our computations designed to demonstrate a method. The size of the task can be seen by noting that for every coefficient included in the adjustment process, 64800 elements of the B_ℓ matrix must be computed, stored and manipulated.

The Data

As implied in the introduction we intend to use three data sources for the combination solution. The first data source are the GEM 9 potnetial coefficients and their standard deviations given by Lerch et al. (1977). This set is complete to degree 20 with additional higher degree coefficients. Not all coefficients of this set will be used in our final solution. We also note that the weight matrix, P_x, for these coefficients will be regarded as a diagonal matrix based on the standard deviations given by Lerch (ibid.).

The next data source is the set of $1^\circ \times 1^\circ$ anomalies that will have to be used in equation (2). To form this data set we merged our most recent ter-

restrial $1^\circ \times 1^\circ$ data set (the June 78 update, Rapp (1978a)), with the anomalies derived from the altimeter data (ibid.). The terrestrial set contained 39405 anomalies and the altimeter set contained 29478 anomalies. The merger of these two data sets took place by replacing all oceanic terrestrial anomaly estimates with the altimeter derived anomalies where available. The total number of anomalies in this combined data set is 50650 with 28176 values based on the altimeter data. A plot of this data is shown in Figure 1. For the remaining 14150 anomalies (needed to complete a global $1^\circ \times 1^\circ$ field) we let the anomaly be zero with a standard deviation of ± 30 mgals which is approximately the square root of the variance of the $1^\circ \times 1^\circ$ anomalies.

In the anomaly merger it was necessary for us to subtract 0.87 mgals from the altimeter derived anomalies as they referred to a gravity formula without atmosphere while the terrestrial anomalies referred to the gravity formula of the Geodetic Reference System 1967 in which the kM value includes the mass of the atmosphere.

Computer Timing of the Various Solutions

A solution of the type proposed here is a costly one in terms of actual computer time and space requirements. The job can be broken down into the following steps:

1. Observation equation formation, specifically the elements of the B_ℓ matrix as given in equation (13).

2. The time for the evaluation of the potential coefficients implied from the input anomalies (i.e. equation (2)).

3. The formation of the M matrix, i.e. equation (10).

4. The inversion of the M matrix.

5. The inversion of the inverted M matrix after the P_x has been added (see equation (15)).

6. The solution vector computation after the inversions have been completed and the adjusted potential coefficients.

7. The evaluation of the anomaly residuals (equation (9)) and the adjusted anomalies.

A number of trial solutions were made for checking and timing purposes. The different solutions primarily depended on the maximum degree for which the input coefficients were to be adjusted. Timings for these steps are given in Table 1 where the runs have been made on a Amdahl 470V/6-II. The timing for step 6 has been omitted as the value for degree 12 was only 0.22 secs.

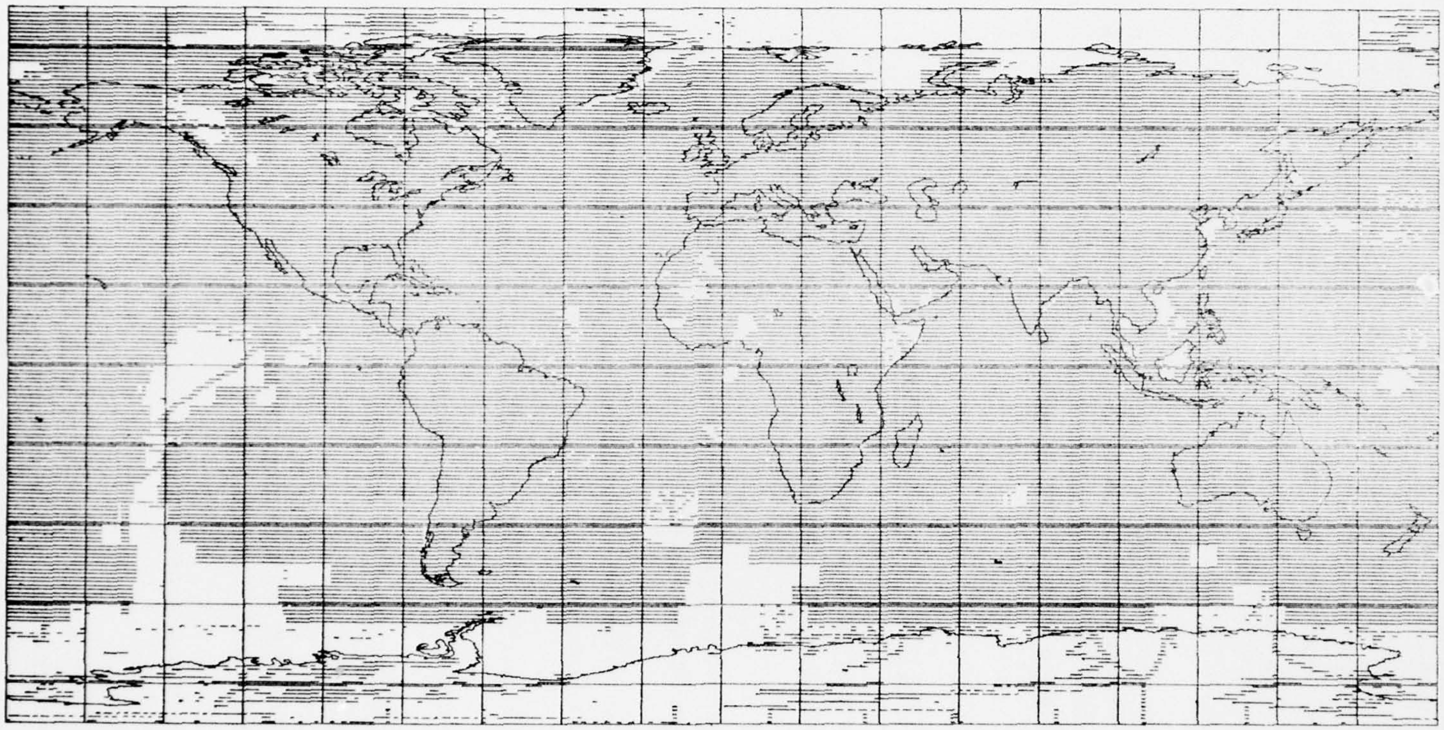

Figure 1. Location of 50650 $1^\circ \times 1^\circ$ Anomalies from Combined Terrestrial-Altimeter Data.

Table 1. Actual Running Times for Combination Solutions Using $1^\circ \times 1^\circ$ Anomalies.

Maximum Degree	Number of Unknowns	Step (see text) units = seconds				
		1 and 2	3	4	5	7
2	9	22	13	.11	.16	8
4	25	32	62	.14	.18	11
6	49	47	219	.25	.30	15
8	81	65	544	.75	.82	61
12	169	113	2302	5.78	5.96	77

The maximum degree for which adjusted coefficients were found for this report was degree 12. If we had gone to include the complete set of GEM 9 coefficients to degree 20 the solution time was estimated to be six hours which was beyond our capability. In addition the storage requirements for the observation equations are quite large. For example, solutions were tried that failed because two 2400 reel tapes had been filled up with information and more tape space was needed.

Solutions and Analysis

Two solutions were made for this study that merit discussion. The first solution adjusted the coefficients to degree 8 and the second solution adjusted the coefficients to degree 12. In both cases the (0,0), and (1,0) terms were forced to zero by specifying an a priori weight of ± 0.01 mgal. The immediate results of each solution were an adjusted set of potential coefficients and an adjusted set of 64800 mean $1^\circ \times 1^\circ$ anomalies. As remarked earlier these adjusted anomalies could be developed into potential coefficients using equation (2).

We now will compare these solutions in several ways. First we developed the adjusted anomalies into potential coefficients to degree 30 and compared these coefficients to the corresponding coefficients in the GEM 9 solution. The differences were expressed in terms of root mean square (RMS) coefficient differences, percentage differences ((P.C. set - GEM 9)/ GEM 9 in %), RMS undulation differences, and RMS anomaly differences. The results are given in Table 2.

Table 2. Comparison of Potential Coefficients Implied by Adjusted $1^\circ \times 1^\circ$ Anomalies for Solutions to Degree 8 and 12 with the GEM 9 Potential Coefficients.

Degree	Coefficient Difference ($\times 10^8$)		Percentage Difference		Undulation Difference (cm)		Anomaly Difference (mgals)	
	8	12	8	12	8	12	8	12
2	.08	.08	.1	.1	1	1	.0	.0
3	.9	.9	.8	.8	16	16	.1	.1
4	.4	.4	.8	.9	8	8	.0	.0
5	1.8	1.9	5	5	38	40	.2	.2
6	2.1	2.1	8	8	48	48	.4	.4
7	2.6	2.3	13	12	64	57	.6	.5
8	2.6	2.4	21	20	67	62	.7	.7
9	3.7	2.6	37	27	102	74	1.3	.9
10	4.0	3.0	51	38	117	87	1.6	1.2
11	3.6	3.0	66	54	111	91	1.7	1.4
12	1.9	1.3	53	37	59	41	1.0	.7
13	2.2	2.1	52	59	72	69	1.3	1.3
14	2.7	2.6	94	90	94	91	1.9	1.8
15	2.1	2.1	90	88	76	75	1.6	1.6
16	2.1	2.0	114	112	76	74	1.8	1.7
17	1.7	1.7	110	108	64	63	1.6	1.5
18	1.5	1.4	81	79	57	55	1.5	1.4
19	1.3	1.3	86	82	53	52	1.5	1.4
20	1.3	1.3	99	100	51	52	1.5	1.5
21	1.3	1.3	132	134	46	47	1.4	1.4
22	1.2	1.3	105	107	44	45	1.4	1.4
23	1.1	1.1	102	101	23	23	0.8	0.8
24	.9	0.9	120	121	20	20	0.7	0.7
25	.5	0.5	50	48	12	11	0.4	0.4
26	.9	0.9	172	168	13	13	0.5	0.5
27	1.0	1.0	90	89	19	18	0.7	0.7
28	.8	0.8	76	76	17	18	0.7	0.7
29	1.1	1.1	102	101	19	19	0.8	0.8
30	2.8	2.8	100	101	25	25	1.1	1.1
	2.1*	1.8*	70$^+$	68$^+$	313^{++}	279^{++}	6.2^{++}	5.8^{++}

* RMS Coefficient Difference

$^+$ Mean Difference

$^{++}$ Overall RMS Difference

We see from this table that for the coefficients adjusted in the degree 8 solutions the maximum RMS difference (for the undulations) by degree is 67 cm

(at degree 8) with the corresponding value for the degree 12 solution being 91 cm at degree 11. The coefficients just beyond degree 8 (say 10 thru 12) of the degree 8 solution disagree with the GEM 9 coefficients more than the coefficients of the degree 12 solution. However beyond that the coefficient differences are essentially the same indicating that the additional coefficients solved for in the degree 12 solution do not play a strong role in the coefficients at the higher degrees.

We have also compared the two coefficient sets to degree 30 of the degree 8 and degree 12 solution. Over the whole set the RMS coefficient difference was $\pm 0.0037 \times 10^{-6}$, the average percentage difference was 7%, the RMS undulation difference was 74 cm and the RMS anomaly difference was 1.1 mgals. At degree 8 the undulation difference was 11 cm increasing to 40 cm at degree 9. At degree 12 the difference was down to 26 cm and at degree 30 it was 2 cm. These results again indicate that the higher degree terms are not strongly influenced by the maximum degree of the adjusted coefficient set.

We have also examined the adjusted $1^\circ \times 1^\circ$ anomalies from the two solutions. The root mean square difference of the two anomaly fields was ± 1.4 mgals with the maximum difference being 78 mgals. For the degree 8 solution the RMS (area averaged) residual was ± 3.1 mgals while the corresponding value for the degree 12 solution was ± 3.6 mgals. The largest residual in the degree 8 solution was 44 mgals while for the degree 12 solution it was 93 mgals. These large residuals are applied to anomalies having the highest standard deviations in the combined starting set. These values could reach ± 81 mgals. It is clear from examination of the residuals that the largest residuals occur for those blocks having the largest standard deviation. To demonstrate this we have computed the RMS $1^\circ \times 1^\circ$ residual as a function of the standard deviations assigned to the anomalies. These results for both the degree 8 and degree 12 solution are given in Table 3.

Table 3. RMS $1^\circ \times 1^\circ$ Anomaly Residuals as a Function of the Anomaly Stnadard Deviation.

Anomaly Standard Deviation Range (mgals)	RMS Residual (mgals)	
	$\ell = 8$	$\ell = 12$
1 to 5	.2	.3
6 to 10	.6	.7
11 to 15	1.8	2.1
16 to 20	3.0	3.6
21 to 25	4.3	5.1
26 to 30	7.7	7.9
31 to 35	15.5	27.9

In effect what appears to happen is that most of the anomaly correction is put into blocks having the poorest accuracy. This has positive and negative aspects.

For additional analysis we computed potential coefficients to degree 60 from the adjusted anomalies of the degree 12 adjustment. With this set we computed the anomaly degree variances defined by:

$$c_\ell = \gamma^2 \, (\ell - 1)^2 \sum_{m=0}^{\ell} (\overline{C}_{\ell m}^2 + \overline{S}_{\ell m}^2) \tag{17}$$

where the $\overline{C}_{\ell,0}$ (ℓ even) coefficients were referred to an ellipsoid whose flattening was 1/298.247. Such values are shown in Figure 2 along with the GEM 9 values (to $\ell = 20$) and values from 5° mean anomalies based on a previous terrestrial data set (Rapp, 1977b), and those implied by the $10^{-5}/\ell^2$ rule of Kaula. Between degrees 2 and 12 the GEM 9 degree variances and those from the adjusted 1° anomalies agree fairly well as would be expected. Between degree 13 and 20 the adjusted anomalies imply somewhat more power than the GEM 9 coefficients. Between degrees 22 and 35 the degree variances from the 1° data agree well with the previously determined values from 5° anomalies; from degree 36 to degree 52 the 5° results are consistently larger than the 1° results. This occurence may be due to the unwarrented application of the smoothing operator at this high degree for the 5° anomalies.

Another way to look at the potential coefficients implied by the 1° x 1° adjusted anomalies is to look at the root mean square potential coefficient variation by degree (Rapp, 1977b). Such variations are shown in Figure 3 for the $10^{-5}/\ell^2$ rule; the values implied by the adjusted 1° x 1° anomalies; and values computed by Wagner (1978) from a spectral analysis of Geos-3 altimeter arcs. We see excellent agreement with the Wagner results and that obtained from our adjusted 1° x 1° anomaly field. Again it is clear that the $10^{-5}/\ell^2$ rule gives too large coefficients out to about degree 60. This fact is also clear from Figure 4 of Rapp (1977b) when comparisons were made with results from the analysis of the 5° terrestrial anomalies. Additional conclusions in this area await the development of the global 1° x 1° field into a higher degree spherical harmonic expansion.

Summary and Conclusions

This report has described and implemented a procedure that can be used to combine satellite derived potential coefficients, altimeter derived 1° x 1° gravity anomalies, and terrestrial 1° x 1° anomalies. This combination takes place in an adjustment process considering the accuracies of all data types involved. The specific results will include a set of adjusted potential coefficients to a ℓ_{max} plus an adjusted set of 1° x 1° anomalies that are exactly consistent with the adjusted potential coefficients and that still retain the high frequency infor-

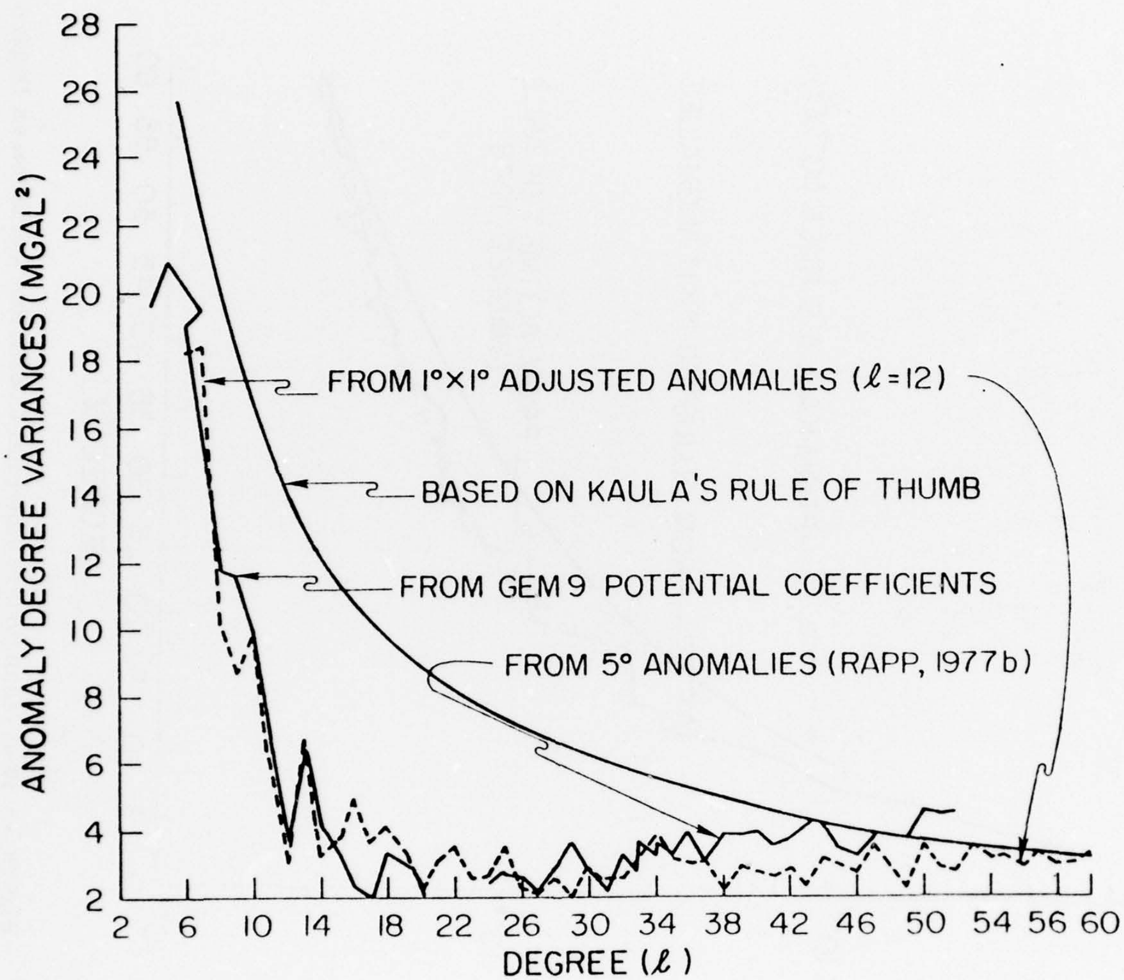

Figure 2. Anomaly Degree Variances vs Degree (ℓ)

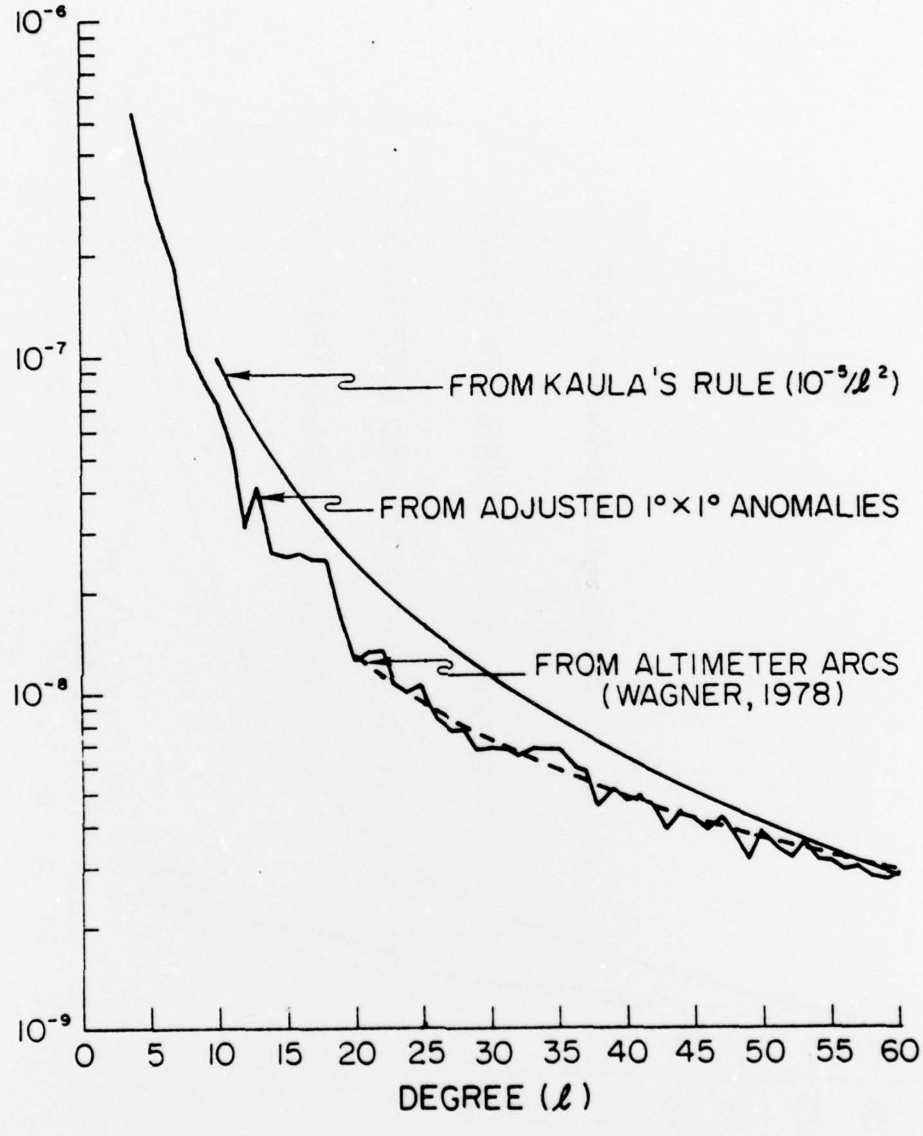

Figure 3. Root Mean Square Potential Coefficient Variation vs Degree (ℓ)

mation inherent in the $1^\circ \times 1^\circ$ data independent of the chosen ℓ_{max}.

For the most general applications one would take the complete coefficient set which may be given to degree 20 plus additional terms, and carry out the combination solution. The adjusted values will thus represent the "best" estimate of the quantities involved considering the data and standard deviations of that data. For this report we carried out a combination solution with the ℓ_{max} of the satellite potential coefficient set (GEM9) being 8 or 12. After the degree 12 solution we developed the adjusted anomalies into potential coefficients to degree 60. With more efficient computer programs this development could easily have been taken to degree 180.

A usual product of this type of investigation is a set of potential coefficients. They are not given in this report because of space reasons but they are available from the author or AFGL (LW), Hanscom AFB, Massachusetts 01731. In addition another specific result is the adjusted $1^\circ \times 1^\circ$ anomalies. Again these 64800 values are not given in this report because of space reasons.

In comparing the degree 8 and degree 12 solutions we found the maximum differences in the coefficients implied by the adjusted anomalies to occur at degree 9 thru 14 after which the coefficients are quite close. (The percentage difference at degree 8 was 3%; at degree 10, 16%; at degree 12, 26%; at degree 14, 11%; at degree 16, 8%; and at degree 30, 5%). This implies that the higher degree terms are not sensitive to the ℓ_{max} used in the actual adjustment process.

We have computed the anomaly degree variances implied by these coefficients as well as the root mean square coefficient variation. They were compared to values from other sources (see Figures 2 and 3). We saw that the $10^{-5}/\ell^2$ rule give variations too large with respect to our data (except that at degree 60 there was fairly good agreement). We compared the spectra implied by our adjusted anomalies to that found by Wagner (1978) from the analysis of altimeter tracks and found excellent agreement with Wagner's results falling as an average thru our results.

A number of things can be done to improve the solution described in this report. We might extend the ℓ_{max} to a higher degree. Incorporating such data from satellite derived potential coefficients would enhance the resulting coefficients for satellite orbit computations. However we found that the estimated time for a solution to degree 20 would take 6 hours on an Amdahl 470/6-II. This time and space requirements are beyond our capability at the present time.

In addition we should consider the effect of correction terms due to the spherical approximation, in equation (2), the neglect of the topography in computing $\overline{\Delta g}_1$, and the effect of the atmosphere. These effects are discussed in detail in Rapp (1977b) when 5° anomalies have been considered. The most critical effect seems to be the terrain but the errors in neglecting it only reached an estimated 6% at degree 36 based on our previous 5° analysis.

The real advantage of the solution described here, above the combination itself advantage, is that we have obtained a set of $1^\circ \times 1^\circ$ anomalies consistent with the adjusted coefficients that also retain the high frequency information inherent in the $1^\circ \times 1^\circ$ anomalies. Such anomalies could be used for orbit computation, geophysical interpretation, and for geoid and deflections of the vertical computation.

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